Rewarding Learning

ADVANCED

General Certificate of Education

2023

Further Mathematics

Assessment Unit A2 2

assessing

Applied Mathematics

AFM21

[AFM21]

MONDAY 5 JUNE, AFTERNOON

TIME

2 hours 15 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided and on the Section D Supplementary Answer Booklet (if applicable).
You must answer **all** questions from sections A and B **or** A and C **or** A and D **or** C and D.
You should spend equal time on each of the two sections.
Candidates taking Section D should use the insert provided and attach to your Answer Booklet using the treasury tag provided.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 150.
The total mark for each section of this paper is 75.
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
Answers should include diagrams where appropriate and marks may be awarded for them.
Take $g = 9.8\,\mathrm{m\,s^{-2}}$, unless specified otherwise.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

SECTION A Mechanics 1

Answer all six questions in this section.

1 A car is moving in a horizontal circle of radius 50 metres around a banked curve inclined at $10°$ to the horizontal, as shown in **Fig. 1** below.

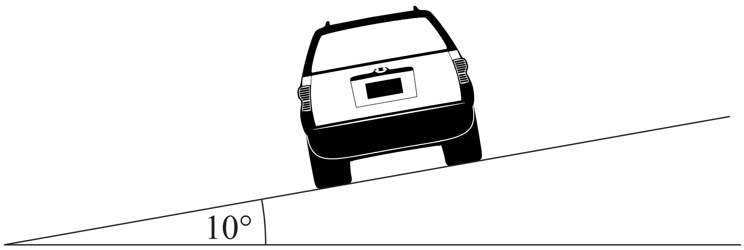

Fig. 1

The coefficient of friction between the wheels and the road is 0.6

Find the maximum speed at which the car could negotiate the bend without slipping. (You may assume that the car does not topple.) [9]

2 A seconds pendulum is correct at location A where the acceleration due to gravity is $9.81\,\mathrm{m\,s^{-2}}$

(i) Find the length of the pendulum. [2]

The pendulum is moved to location B where the acceleration due to gravity is $9.92\,\mathrm{m\,s^{-2}}$

(ii) Find how many more oscillations per day the pendulum makes at B. [6]

(iii) Find the required percentage increase in the original length of the pendulum so that it acts as a seconds pendulum at B. [4]

3 The vertical motion of the water level in Kilkeel harbour can be modelled by Simple Harmonic Motion.

Low tide is at 2.00 am with a depth of 1.5 metres of water in the harbour.

High tide is at 8.30 am with a water depth of 4 metres.

(i) Find the amplitude and period of this motion. [4]

A fishing boat requires a water depth of 1.9 metres in order to leave the harbour.

(ii) Find the earliest time after low tide at which the boat can leave the harbour. [6]

4 A framework of 5 identical light pin-jointed rods is held in a vertical plane as shown in **Fig. 2** below.

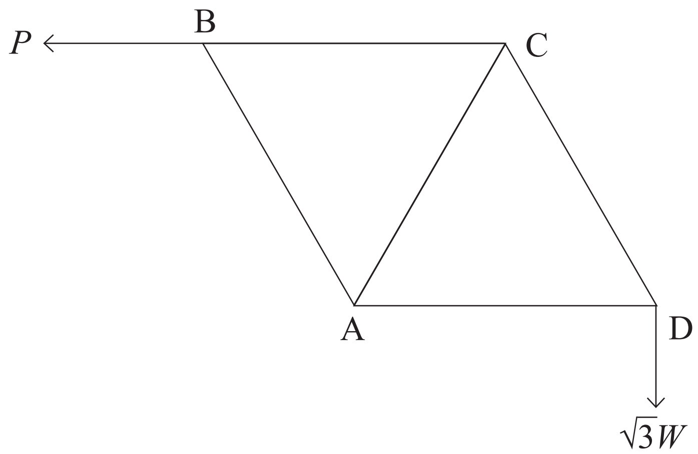

Fig. 2

The framework is freely hinged at A and a particle of weight $\sqrt{3}W$ newtons is suspended from D.

A horizontal force of P newtons is applied at B.

The force P holds the framework in equilibrium with BC and AD horizontal.

(i) Show that $P = 2W$ [3]

(ii) Find, in terms of W, the forces in each of the rods AB, BC, CD, AC and AD. [8]

(iii) State which of these rods could be removed without disturbing the equilibrium of the system. [1]

5 A particle P of mass 0.16 kg is attached to the end of a light spring.

The other end of the spring is attached to a fixed point O on a smooth horizontal table, as shown in **Fig. 3** below.

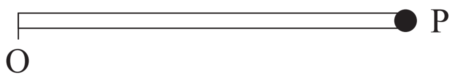

Fig. 3

The system can be modelled as a perfect spring and a perfect dashpot.

The spring has natural length 0.25 metres and modulus of elasticity 0.36 newtons.

The dashpot has damping constant 0.96 Ns m^{-1}

Initially the spring is stretched by 0.04 m and P is released from rest.

(i) Show that the motion of P is given by

$$\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 9x = 0$$

where x metres is the displacement of P from the equilibrium position at time t seconds. [6]

(ii) Find the particular solution of the equation of motion. [8]

(iii) Determine whether the system is critically, under or over damped. [2]

6 An interior designer makes a pendant which consists of 6 light uniform rods, each of length $2a$ metres, as shown in **Fig. 4** below.

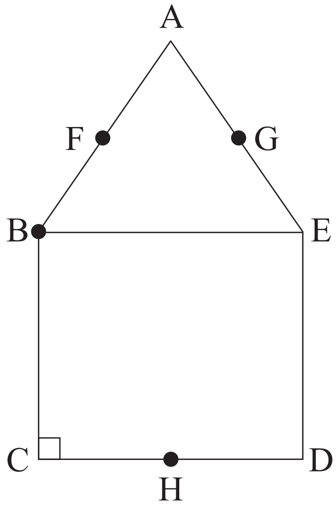

Fig. 4

Angle BCD = 90°

The points F, G and H are the midpoints of AB, AE and CD respectively.

Glass beads with masses (in kg) of $4m$, m, $3m$ and $2m$ are fixed at the points B, F, G and H respectively.

(i) Find the perpendicular distance of the centre of mass of the pendant from each of AH and BE. [10]

The pendant is freely suspended from A.

(ii) Find the horizontal force which must be applied at D to ensure that the pendant remains in equilibrium with CD horizontal. [3]

The designer instead decides to add another glass bead at E so that no extra force is required for the pendant to remain in equilibrium with CD horizontal.

(iii) Find the mass of the bead at E. [3]

SECTION B Mechanics 2

Answer all five questions in this section.

1 A particle of mass m kg falls vertically from rest under gravity against a resistive force of $5mv$ newtons, where $v\,\mathrm{m\,s^{-1}}$ is its velocity at time t seconds.

Find the time taken to reach a speed of $1.9\,\mathrm{m\,s^{-1}}$ [9]

2 At time t seconds a particle has a velocity $\mathbf{v}\,\mathrm{m\,s^{-1}}$ given by

$$\mathbf{v} = 6\sin 3t\,\mathbf{i} + 5\cos t\,\mathbf{j} - 6\cos 3t\,\mathbf{k}$$

(i) Find the acceleration of the particle when $t = \frac{\pi}{6}$ [4]

When $t = 0$ the displacement of the particle from the origin O is $-2\mathbf{i}$ metres.

(ii) Find the maximum distance of the particle from O and identify the first time at which this occurs. [8]

3 Take **i** and **j** to be unit vectors in the positive directions of the Ox and Oy axes respectively.

Three forces, $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$, act at the points (2, 3), (1, –2) and (–3, 2) respectively, as shown in **Fig. 1** below.

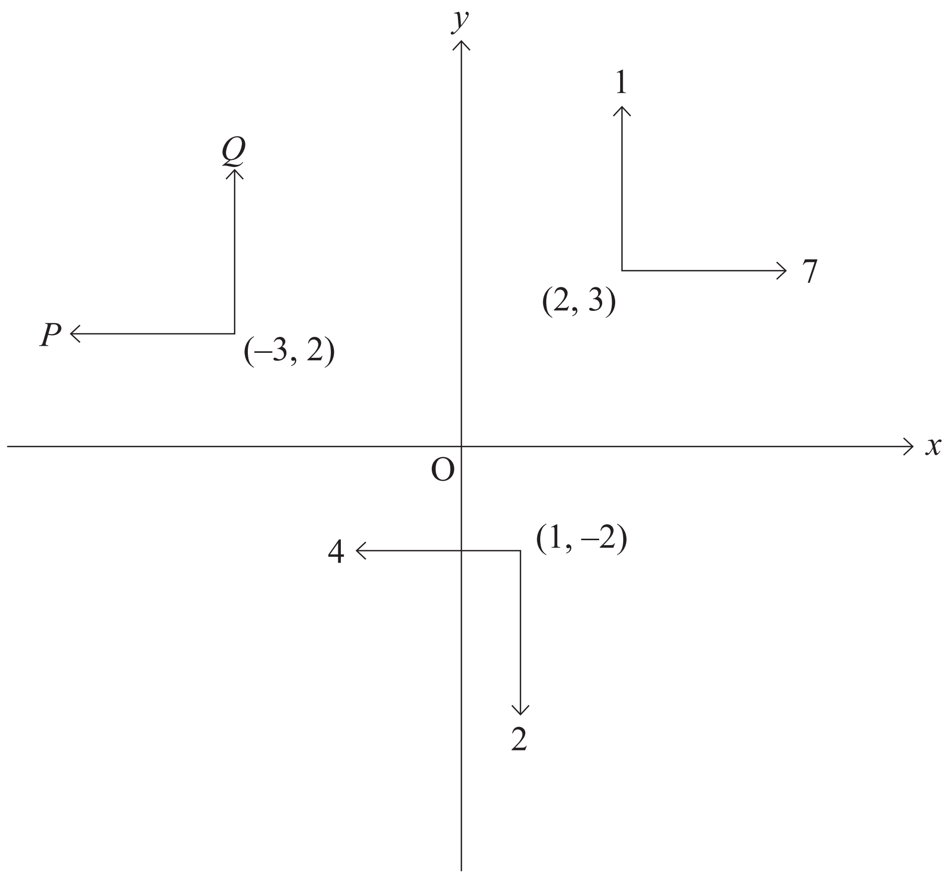

Fig. 1

$$\mathbf{F}_1 = (7\mathbf{i} + \mathbf{j})\,\mathrm{N}$$
$$\mathbf{F}_2 = (-4\mathbf{i} - 2\mathbf{j})\,\mathrm{N}$$
$$\mathbf{F}_3 = (-P\mathbf{i} + Q\mathbf{j})\,\mathrm{N}$$

where P, Q are constants and the unit of distance is metres.

(i) Find the resultant force in terms of P and Q. [2]

(ii) Find the sum of the moments of the three forces about the origin, in terms of P and Q. [3]

(iii) Show that this system of forces cannot be in equilibrium. [5]

(iv) If this system of forces is equivalent to a single force $\mathbf{F} = \alpha\,(3\mathbf{i} + 2\mathbf{j})$ newtons acting at the point (2, 1), find the value of α. [7]

4 Three identical spheres, A, B and C, each of mass m kg, lie at rest in a straight line on a smooth horizontal table as shown in **Fig. 2** below.

Fig. 2

A is projected towards B with a speed of $u\,\mathrm{m\,s^{-1}}$ and collides directly with B.

The coefficient of restitution between A and B is e.

(i) Show that the velocity of B after the collision is $\frac{1}{2}u(1+e)\,\mathrm{m\,s^{-1}}$ [6]

B then collides directly with C.

The coefficient of restitution between B and C is e.

After this collision, the direction of motion of B is unchanged and it now moves with a speed of $\frac{6}{25}u\,\mathrm{m\,s^{-1}}$

(ii) Show that $e = \frac{1}{5}$ [4]

(iii) Find, in terms of m and u, the total energy loss as a result of the two collisions. [8]

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5 The centre of mass of a uniform solid cone with base radius r cm and height h cm lies d cm from the base along the cone's line of symmetry.

(i) Show that $d = \frac{1}{4}h$ [7]

Fig. 3 below shows a frustum of a uniform solid cone.

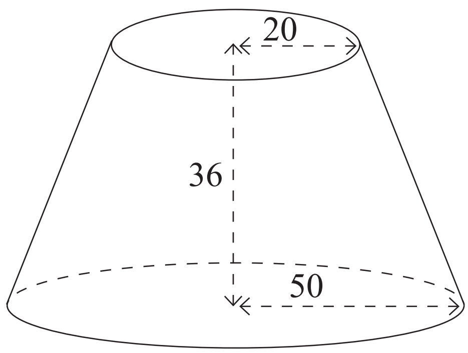

Fig. 3

The radius of its upper circular face is 20 cm.

The radius of its circular base is 50 cm.

The perpendicular distance between these two circular faces is 36 cm.

(ii) Find the perpendicular distance of the frustum's centre of mass from its base. [8]

Fig. 4 below shows the frustum placed on a rough plane inclined at $\theta°$ to the horizontal.

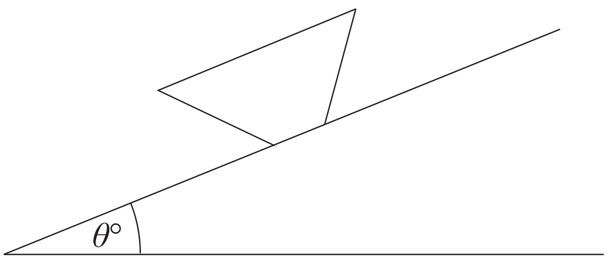

Fig. 4

(iii) Find the value of θ if the frustum is about to topple.
(It may be assumed that the plane is sufficiently rough to prevent sliding.) [4]

SECTION C Statistics

Answer all five questions in this section.

1 Two independent random variables S and T are such that $S \sim \mathrm{N}(36, 7)$ and $T \sim \mathrm{N}(42, 3)$.

S_1 and S_2 are independent observations of the variable S.

T_1 and T_2 are independent observations of the variable T.

(i) Find $\mathrm{E}(S_1 + 3S_2 - 4T_1 + T_2)$ and $\mathrm{Var}\,(S_1 + 3S_2 - 4T_1 + T_2)$. [5]

(ii) Hence find $\mathrm{P}(S_1 + 3S_2 < 4T_1 - T_2)$. [3]

2 A random variable X has mean 100 and standard deviation σ.
49 independent observations of X are recorded.

The probability that the mean of the 49 observations is less than 98 is 0.25

(i) Find an estimate for the value of σ. [8]

(ii) Hence find the standard error of the mean. [1]

(iii) What theorem have you used in part **(i)** and why were you able to use it? [2]

3 A medical researcher is investigating the effects of smoking on lung function.

From a national database she randomly selects 12 pairs of identical adult twins.

In each pair one twin has smoked regularly for at least five years, whereas the other twin has never smoked.

In order to measure lung function, the researcher conducts a suitable spirometry test on each twin and records the result of the test.

The test result is a single value, with a lower value indicating reduced lung function.

The results for the 12 pairs of twins in the sample are shown in **Table 1** below.

Table 1

Twin pair	Non-smoking twin test result	Smoking twin test result
1	102	97
2	101	92
3	113	101
4	84	90
5	73	79
6	93	77
7	88	74
8	100	82
9	80	81
10	91	78
11	85	64
12	91	84

(i) Carry out a suitable test, at a 0.5% level of significance, to test the claim that those smoking regularly for at least five years have reduced lung function, compared to those who have never smoked. [16]

(ii) State one other difference in each pair that the researcher might consider when selecting her sample. [1]

(iii) State why it was necessary for the researcher to use identical twins for the purposes of the investigation. [2]

4 At a national schools' long jump competition, the distances, x cm, jumped by pupils in the Under-16 event are Normally distributed with standard deviation 12 cm.

The results, in cm, for a random sample of seven pupils are listed below.

508, 516, 528, 530, 532, 537, 545

(i) Find a 99% confidence interval for the population mean based on this sample of seven pupils. [6]

The competition organisers require the 99% confidence interval for the population mean to have a total width of less than 10 cm.

(ii) Find the smallest size of sample that would satisfy this condition. [4]

At the same competition, the distances, y cm, jumped by Under-18 competitors were recorded.

A sample of 40 pupils was randomly selected.

This sample of size 40 gave the following summary statistics:

$$\sum y = 21\,200 \qquad \sum y^2 = 11\,242\,222$$

(iii) Find unbiased estimates of the mean and variance of distances jumped. [3]

(iv) Find a 95% confidence interval for the population mean based on this sample of 40 pupils. [4]

The competition organisers use the result from **(iv)** to claim that 95% of all the Under-18 results lie within this interval.

(v) Explain why this is not necessarily true. [1]

5 As part of their annual review, road traffic police officers must keep a record of the number of speeding fines that they issue per week, x, over the course of one year.

The results for one particular officer are shown in **Table 2** below.

Table 2

Number of speeding fines (x)	0	1	2	3	4	5
Number of weeks with x fines issued	5	4	16	16	9	2

The expected frequencies of x, as calculated from a Poisson distribution with mean estimated from the observed results, are shown in **Table 3** below.

The expected frequencies are given to one decimal place.

Table 3

x	0	1	2	3	4	5 or more
Expected number of weeks	4.3	10.7	13.3	a	6.9	b

(i) Show that $a = 11.1$ and $b = 5.7$ [7]

(ii) Carry out a suitable test, at the 5% level, to assess the suitability of this Poisson distribution as a model for the data. [11]

(iii) The chi-squared statistic for the results of a second officer was found to be 0.01
Comment on this value. [1]

SECTION D Discrete and Decision Mathematics

Answer all six questions in this section.

Note: There is a **Supplementary Answer Booklet** for questions **1(a)(ii)**, **2(ii)**, **6(ii)** and **(iii)**. Please attach this to your answer booklet using the treasury tag provided.

1 **(a)** The complete bipartite graph $K_{3,3}$ and the complete graph on 4 vertices K_4 are shown in **Fig. 1** below.

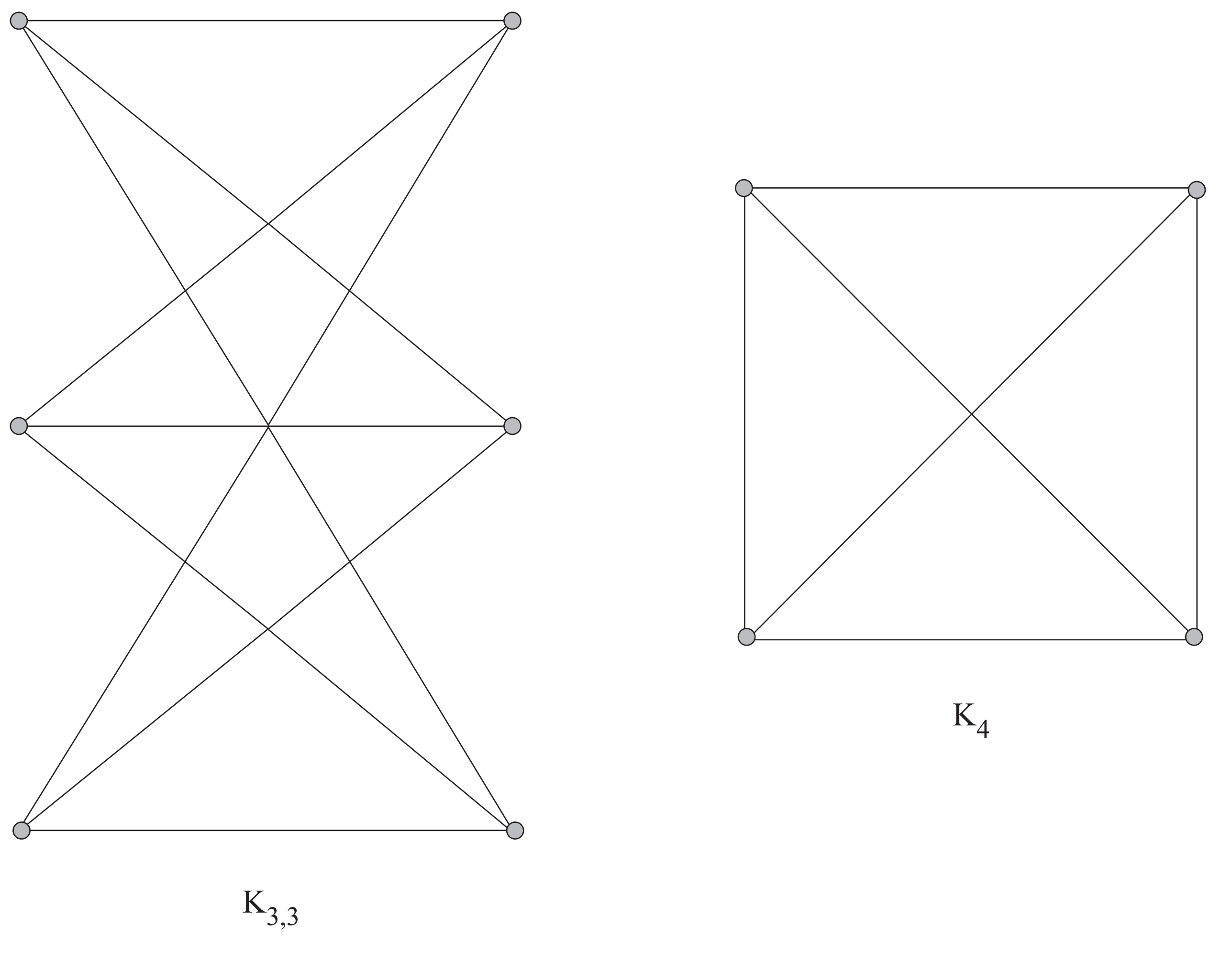

Fig. 1

(i) For each of the above graphs, write down the minimum number of colours needed for a vertex-colouring. [2]

(ii) Using the diagrams in the **Supplementary Answer Booklet** and denoting different colours by the integers 1, 2, … design a minimum edge-colouring for each of the above graphs. [3]

(b) Every week a Physical Education class of 16 girls has to form 8 pairs for a game.

In week one, the first half of the class pick partners. **Table 1** below gives the partners each would be happy with.

Table 1

Pupil	Aoife	Beatrice	Clodagh	Debbie	Eunice	Felicity	Gwen	Harriet
Possible partners	Janice Katherine	Meaghan Olive	Katherine Laura	Niamh Patricia	Janice Katherine	Meaghan Niamh	Janice Laura	Patricia Quetzal

(i) Prove, using Hall's Marriage Theorem, that a complete matching is not possible for these choices. [3]

In week two, the second half of the class pick partners. Their preferences are listed in **Table 2** below.

Table 2

Pupil	Janice	Katherine	Laura	Meaghan	Niamh	Olive	Patricia	Quetzal
Possible partners	Aoife Clodagh	Beatrice Eunice	Debbie Felicity	Aoife Beatrice	Clodagh Harriet	Aoife Gwen	Beatrice Debbie	Felicity Harriet

(ii) Find a complete matching for these preferences. [2]

(c) **Fig. 2** below shows the graph G on vertices a, b, c, d and e.
A weight is assigned to each edge.

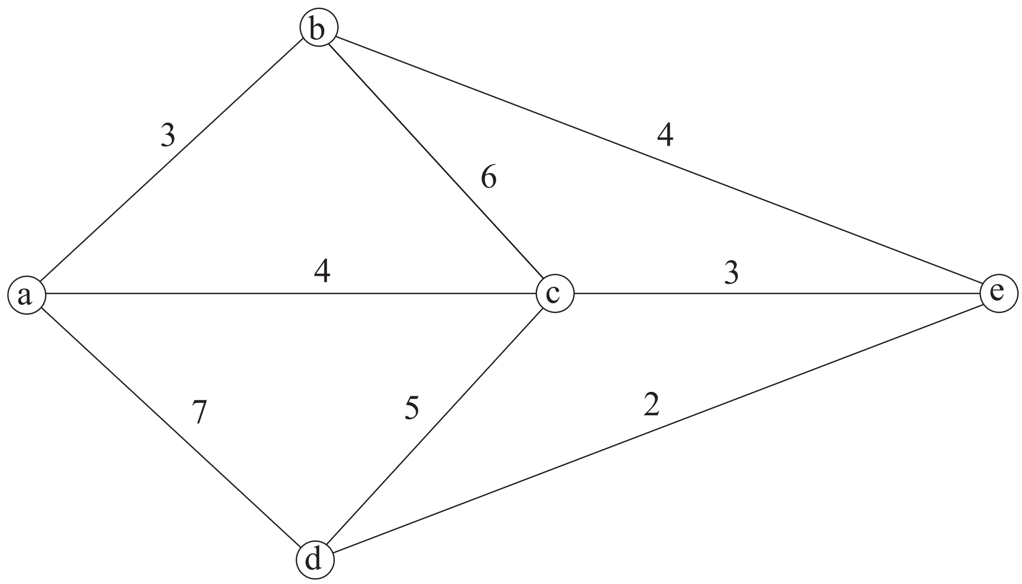

Fig. 2

(i) Using the Nearest Neighbour Algorithm, starting at vertex a, find a Hamiltonian path in the graph G. [2]

(ii) Find, by inspection, a Hamiltonian path with minimum weight.

(Note that it is not necessary to start at vertex a.) [1]

(iii) What characteristic of the Nearest Neighbour Algorithm means it sometimes fails to find a Hamiltonian path with minimum weight? [1]

2 A precedence table is shown in **Table 3** below for the preparations for an art exhibition.

The numbers are times in hours.

Table 3

Activity	Predecessor	Optimal time	Normal time	Pessimistic time	Expected time
A	–	6	7	8	7
B	A	5	7	9	7
C	A	6.5	8	9.5	8
D	B, C	10.5	13	15.5	13
E	B, C	3	4	5	
F	D	2.5	3	3.5	
G	D, E	9	12	15	
H	F, G	5	6	7	

(i) Find the expected times for activities E, F, G and H. [2]

(ii) In the **Supplementary Answer Booklet**, complete the PERT chart for the exhibition. [4]

(iii) List the critical path. [1]

(iv) Find the probability that the preparations for the art exhibition will take longer than 48 hours. [7]

3 **Fig. 3** below shows a board.

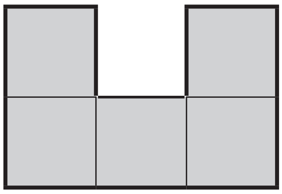

Fig. 3

(i) Show that the rook polynomial for the board above is

$$p + qx + 4x^2$$

where p and q are positive integers to be found. [2]

Fig. 4 below shows a compound board.

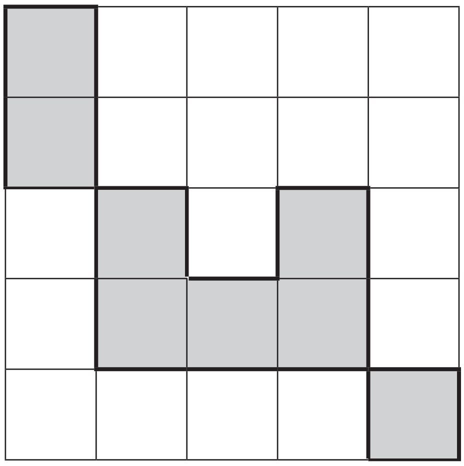

Fig. 4

(ii) Show that the rook polynomial for this compound board is

$$1 + 8x + 21x^2 + 22x^3 + 8x^4$$ [7]

The Vegetable Growers' Association are assigning five jobs to their committee members Pierre, Quentin, Raul, Sigismund and Tomo, subject to the following rules:

Overall	no person can have two jobs
Pierre	will not do jobs 1 or 2
Quentin	will not do jobs 3 or 4
Raul	will not do job 4
Sigismund	will not do jobs 3 or 4
Tomo	will not do job 5

(iii) Using part **(ii)** and the Inclusion Exclusion Principle, determine the number of different ways the jobs may be assigned to committee members. [7]

4 **(a)** The cycle index for the edge symmetries of an octahedron is

$$P_G = \frac{1}{24}\left(x_1^{12} + 3x_2^6 + 6x_1^2x_2^5 + 6x_4^3 + 8x_3^4\right)$$

A jeweller designs a pendant in the form of an octahedral framework.

Each of its 12 bars may be made of one of the four semi-precious materials, jade, quartz, beryl or antimony.

How many different pendants are possible? [3]

(b) The cycle index for the edge symmetries of a tetrahedron is:

$$P_G = \frac{1}{12}\left(x_1^6 + 8x_3^2 + 3x_1^2x_2^2\right)$$

(i) Calculate the pattern inventory for the colouring of the six edges of a tetrahedron using two colours, blue (B) and green (G).

(You need not fully expand the expression.) [3]

A chemist wishes to colour each edge of a model of a tetrahedral molecule, blue or green.

(ii) How many different possible colourings are there with three blue edges and three green edges? [5]

5 The sequence $a_0,\ a_1,\ a_2,\ a_3, \ldots$ has the generating function

$$f(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + \cdots$$

(i) Write down the sequence which has $\dfrac{d}{dt}[f(t)]$ as its generating function. [1]

(ii) By considering the series expansion $\dfrac{1}{1-t} = 1 + t + t^2 + t^3 + t^4 + \cdots$

find the closed form of the generating function for the sequence 1, 2, 3, 4, ⋯ [3]

(iii) Hence find the closed form of the generating function for the even positive integers.

(That is, not the infinite series expansion for the function.) [2]

(iv) By combining two of the above generating functions, derive the closed form of the generating function for the odd positive integers. [3]

6 A linear programming problem is stated as:

Maximise $P = 3x + 2y$ subject to the constraints

$$4x + y \leqslant 16 \quad \text{and}$$

$$2x + 5y \leqslant 26$$

where x and y are non-negative.

(i) Using slack variables, restate the problem in terms of equations in preparation for the use of the simplex method. [3]

(ii) Hence, using the grid in the **Supplementary Answer Booklet**, complete an initial simplex tableau for this problem. [3]

(iii) Using the copies of the grid given in the **Supplementary Answer Booklet**, perform one iteration of the simplex method. Carefully identify the pivot element and rewrite the tableau. [3]

After a further iteration of the simplex method, the final tableau is given in **Table 4** below.

Table 4

x	y	u	v	P	
1	0	$0.2\dot{7}$	$-0.0\dot{5}$	0	3
0	1	$-0.\dot{1}$	$0.\dot{2}$	0	4
0	0	$0.6\dot{1}$	$0.2\dot{7}$	1	17

(iv) Write down the optimal solution from the above tableau. [2]

THIS IS THE END OF THE QUESTION PAPER

Centre Number

Candidate Number

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General Certificate of Education

2023

Further Mathematics

Assessment Unit A2 2

assessing

Applied Mathematics

[AFM21]

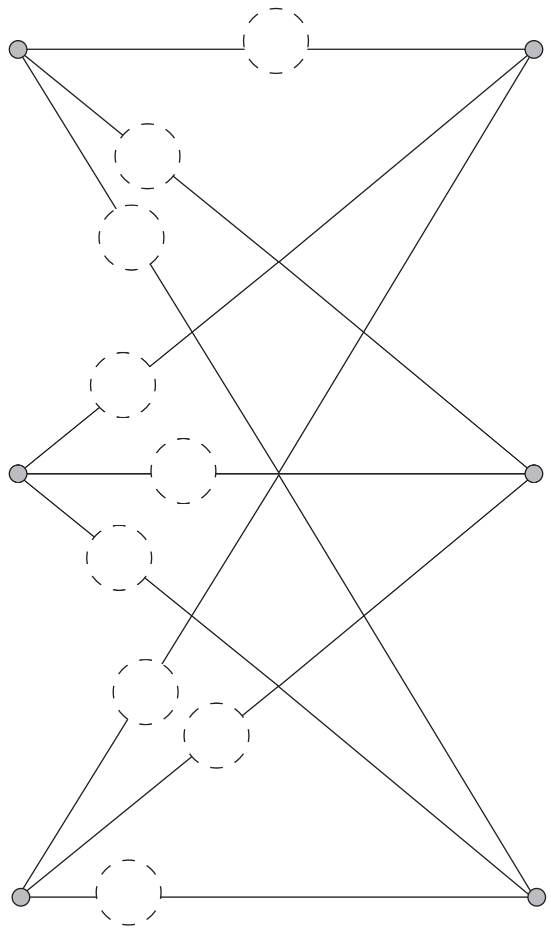
MONDAY 5 JUNE, AFTERNOON

SECTION D

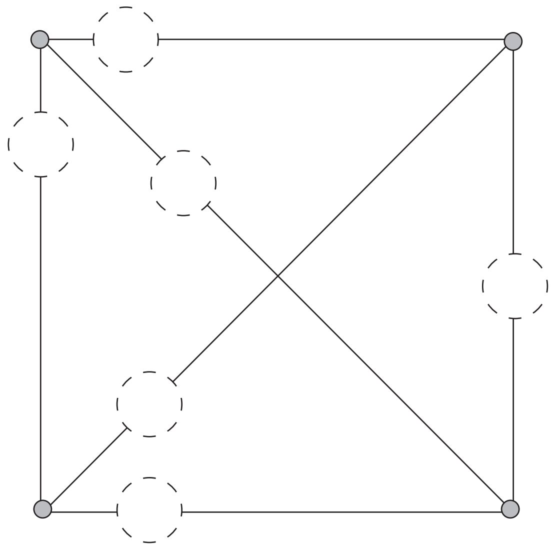
SUPPLEMENTARY ANSWER BOOKLET

1 (a) (ii) Denoting different colours by the integers 1, 2, ... design a minimum edge-colouring for each of the graphs in the diagram below.

You may insert the integers to represent the colours in the dotted circles on each edge.



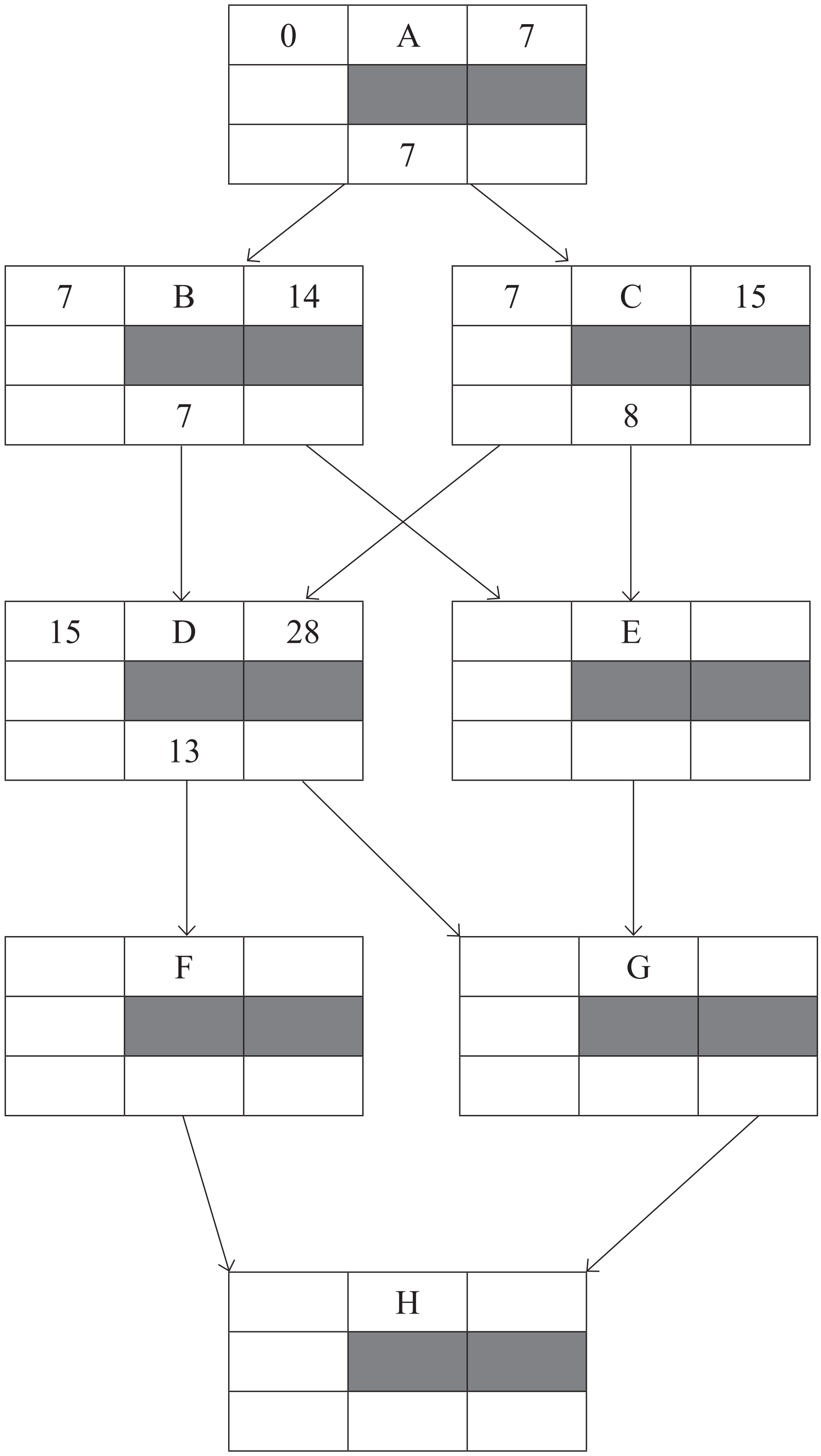
$K_{3,3}$



K_4

Fig. 1

2 **(ii)** Complete the PERT chart for this art exhibition.



Key:

ES	Node	EF
Float		
LS	Expected time	LF

ES = Early start time

LS = Late start time

EF = Early finish time

LF = Late finish time

6 **(ii)**

x	y	u	v	P	

(iii)

x	y	u	v	P	

x	y	u	v	P	

Rewarding Learning

ADVANCED/ADVANCED SUBSIDIARY (A/AS)

General Certificate of Education

Mathematical Formulae and Tables

For use by candidates taking the Advanced Subsidiary and Advanced GCE examinations in Mathematics and Further Mathematics

For use from 2019

Appendix 1

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CONTENTS

PURE MATHEMATICS

Mensuration

Surface area of sphere = $4\pi r^2$
Area of curved surface of cone = $\pi r \times \text{slant height}$

Summations

$$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$$

$$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$$

Arithmetic Series

$u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\left[2a+(n-1)d\right]$$

Geometric Series

$u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$S_\infty = \dfrac{a}{1-r}$ for $|r| < 1$

Binomial Series

$$\binom{n}{r} + \binom{n}{r+1} = \binom{n+1}{r+1}$$

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N})$$

where $\binom{n}{r} = {}^n\mathrm{C}_r = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1.2}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{1.2\ldots r}x^r + \ldots \qquad (|x| < 1, n \in \mathbb{R})$$

Logarithms and exponentials

$$e^{x \ln a} = a^x$$

Complex Numbers

$$\{r(\cos\theta + \mathrm{i}\sin\theta)\}^n = r^n(\cos n\theta + \mathrm{i}\sin n\theta)$$

$$e^{\mathrm{i}\theta} = \cos\theta + \mathrm{i}\sin\theta$$

The roots of $z^n = 1$ are given by $z = e^{\frac{2\pi k \mathrm{i}}{n}}$, for $k = 0, 1, 2, \ldots, n - 1$

Maclaurin's Series

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \ldots + \frac{x^r}{r!}f^{(r)}(0) + \ldots$$

$$e^x = \exp(x) = 1 + x + \frac{x^2}{2!} + \ldots + \frac{x^r}{r!} + \ldots \text{ for all } x$$

$$\ln(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots + (-1)^{r+1}\frac{x^r}{r} + \ldots \quad (-1 < x \leq 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots + (-1)^r \frac{x^{2r+1}}{(2r+1)!} + \ldots \text{ for all } x$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots + (-1)^r \frac{x^{2r}}{(2r)!} + \ldots \text{ for all } x$$

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \ldots (-1)^r \frac{x^{2r+1}}{2r+1} + \ldots \quad (-1 < x < 1)$$

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots + \frac{x^{2r+1}}{(2r+1)!} + \ldots \text{ for all } x$$

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots + \frac{x^{2r}}{(2r)!} + \ldots \text{ for all } x$$

$$\tanh^{-1} x = x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots + \frac{x^{2r+1}}{2r+1} + \ldots \quad (-1 < x < 1)$$

Hyperbolic Functions

$\cosh^2 x - \sinh^2 x = 1$

$\sinh 2x = 2\sinh x \cosh x$

$\cosh 2x = \cosh^2 x + \sinh^2 x$

$\cosh^{-1} x = \ln\left(x + \sqrt{x^2 - 1}\right) \quad (x \geq 1)$

$\sinh^{-1} x = \ln\left(x + \sqrt{x^2 + 1}\right)$

$\tanh^{-1} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right) \quad (|x| < 1)$

Trigonometry

In the triangle ABC: $a^2 = b^2 + c^2 - 2bc \cos \mathrm{A}$

Trigonometric Identities

$\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan (A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad (A \pm B \neq (k + \frac{1}{2})\pi)$

Small angle approximations

$\sin \theta \approx \theta$

$\cos \theta \approx 1 - \dfrac{\theta^2}{2}$

$\tan \theta \approx \theta$

where θ is measured in radians.

Vectors

Vector product: $\mathbf{a}\times\mathbf{b} = |\mathbf{a}||\mathbf{b}|\sin\theta\,\hat{\mathbf{n}} = \begin{vmatrix} \mathbf{i} & a_1 & b_1 \\ \mathbf{j} & a_2 & b_2 \\ \mathbf{k} & a_3 & b_3 \end{vmatrix} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix}$

$$\mathbf{a}.(\mathbf{b}\times\mathbf{c}) = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \mathbf{b}.(\mathbf{c}\times\mathbf{a}) = \mathbf{c}.(\mathbf{a}\times\mathbf{b})$$

If A is the point with position vector **a** = $a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and the direction vector **b** is given by **b** = $b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$, then the straight line through A with direction vector **b** has cartesian equation

$$\frac{x - a_1}{b_1} = \frac{y - a_2}{b_2} = \frac{z - a_3}{b_3} \quad (= \lambda)$$

The plane through A with normal vector **n** = $n_1\mathbf{i} + n_2\mathbf{j} + n_3\mathbf{k}$ has cartesian equation
$n_1x + n_2y + n_3z + d = 0$ where $d = -$ **a.n**

The plane through non-collinear points *A*, *B* and *C* has vector equation
$\mathbf{r} = \mathbf{a} + \lambda(\mathbf{b} - \mathbf{a}) + \mu(\mathbf{c} - \mathbf{a}) = (1 - \lambda - \mu)\mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$

The plane through the point with position vector **a** and parallel to vectors **b** and **c** has equation
$\mathbf{r} = \mathbf{a} + s\mathbf{b} + t\mathbf{c}$

The perpendicular distance of (α, β, γ) from $n_1x + n_2y + n_3z + d = 0$ is $\dfrac{|n_1\alpha + n_2\beta + n_3\gamma + d|}{\sqrt{n_1^2 + n_2^2 + n_3^2}}$

Matrix transformations

Anticlockwise rotation through θ about the origin: $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

Reflection in the line $y = (\tan\theta)\,x$: $\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k \sec^2 kx$
$\dfrac{\mathrm{f}(x)}{\mathrm{g}(x)}$	$\dfrac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{(\mathrm{g}(x))^2}$
$\sin^{-1} x$	$\dfrac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$-\dfrac{1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$\dfrac{1}{1+x^2}$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\operatorname{sech}^2 x$
$\sinh^{-1} x$	$\dfrac{1}{\sqrt{1+x^2}}$
$\cosh^{-1} x$	$\dfrac{1}{\sqrt{x^2-1}}$
$\tanh^{-1} x$	$\dfrac{1}{1-x^2}$

Integration
(+ constant; a > 0 where relevant)

$f(x)$	$\int f(x)\,dx$
$\tan x$	$\ln \lvert\sec x\rvert$
$\cot x$	$\ln \lvert\sin x\rvert$
$\operatorname{cosec} x$	$-\ln \lvert\operatorname{cosec} x + \cot x\rvert = \ln \left\lvert\tan\left(\frac{x}{2}\right)\right\rvert$
$\sec x$	$\ln \lvert\sec x + \tan x\rvert = \ln \left\lvert\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\right\rvert$
$\sec^2 kx$	$\frac{1}{k}\tan kx$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\ln \lvert\cosh x\rvert$
$\frac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1}\left(\frac{x}{a}\right)$, $(\lvert x\rvert < a)$
$\frac{1}{a^2+x^2}$	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$
$\frac{1}{\sqrt{x^2-a^2}}$	$\cosh^{-1}\left(\frac{x}{a}\right)$ or $\ln\left(x+\sqrt{x^2-a^2}\right)$, $(x > a)$
$\frac{1}{\sqrt{a^2+x^2}}$	$\sinh^{-1}\left(\frac{x}{a}\right)$ or $\ln\left(x+\sqrt{x^2+a^2}\right)$
$\frac{1}{a^2-x^2}$	$\frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert = \frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right)$, $(\lvert x\rvert < a)$
$\frac{1}{x^2-a^2}$	$\frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert$, $(\lvert x\rvert > a)$

$$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$$

Area of a Sector

$A = \frac{1}{2}\int r^2\,d\theta$ (polar coordinates)

NUMERICAL MATHEMATICS

Numerical integration

The trapezium rule: $\int_a^b y\,\mathrm{d}x \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \frac{b-a}{n}$

Numerical Solution of Equations

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \frac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

MECHANICS

Motion in a circle

Transverse velocity: $v = r\dot{\theta}$

Transverse acceleration: $\dot{v} = r\ddot{\theta}$

Radial acceleration: $-r\dot{\theta}^2 = -\frac{v^2}{r}$

Centres of Mass

For uniform bodies

Triangular lamina: $\frac{2}{3}$ along median from vertex

Solid hemisphere, radius *r* : $\frac{3}{8}r$ from centre

Hemispherical shell, radius *r* : $\frac{1}{2}r$ from centre

Circular arc, radius *r*, angle at centre 2α: $\frac{r\sin\alpha}{\alpha}$ from centre

Sector of circle, radius *r*, angle at centre 2α: $\frac{2r\sin\alpha}{3\alpha}$ from centre

Solid cone or pyramid of height *h*: $\frac{1}{4}h$ above the base on the line from centre of base to vertex

Conical shell of height *h*: $\frac{1}{3}h$ above the base on the line from centre of base to vertex

Universal law of gravitation

Force = $\frac{Gm_1m_2}{d^2}$

PROBABILITY AND STATISTICS

Probability

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = P(A)P(B|A)$

$$P(A|B) = \frac{P(B \mid A)P(A)}{P(B \mid A)P(A) + P(B \mid A')P(A')}$$

Discrete distributions

For a discrete random variable X taking values x_i with probabilities p_i

Expectation (mean): $E(X) = \mu = \sum_i x_i p_i$

Variance: $\mathrm{Var}(X) = \sigma^2 = \Sigma_i (x_i - \mu)^2 p_i = \sum_i x_i^2 p_i - \mu^2$

For a function $g(X)$: $E(g(X)) = \sum_i g(x_i) p_i$

Standard discrete distributions:

Distribution of X	$P(X = x)$	Mean	Variance
Binomial $B(n, p)$	$\binom{n}{x} p^x (1-p)^{n-x}$	np	$np(1-p)$
Poisson $Po(\lambda)$	$e^{-\lambda} \frac{\lambda^x}{x!}$	λ	λ

Continuous distributions

For a continuous random variable X having probability density function $f(x)$:

Expectation (mean): $E(X) = \mu = \int x\, f(x)\,dx$

Variance: $\mathrm{Var}(X) = \sigma^2 = \int (x-\mu)^2\, f(x)\,dx = \int x^2\, f(x)\,dx - \mu^2$

For a function $g(X)$: $E(g(X)) = \int g(x)\, f(x)\,dx$

Standard continuous distributions

Distribution of X	P.D.F.	Mean	Variance
Uniform (Rectangular) on $[a, b]$	$\frac{1}{b-a}$	$\frac{1}{2}(a+b)$	$\frac{1}{12}(b-a)^2$
Normal $N(\mu, \sigma^2)$	$\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$	μ	σ^2

Expectation algebra

For independent random variables X and Y

$E(XY) = E(X)E(Y)$; $\mathrm{Var}(aX \pm bY) = a^2\,\mathrm{Var}(X) + b^2\,\mathrm{Var}(Y)$

Sampling distributions

For a random sample $x_1, x_2, \ldots, x_n$ of n independent observations from a distribution having mean μ and variance σ^2

$\bar{x}$ is an unbiased estimator of μ, with $\mathrm{Var}(\bar{x}) = \frac{\sigma^2}{n}$

S^2 is an unbiased estimator of σ^2, where $S^2 = \frac{\Sigma(x_i - \bar{x})^2}{n-1}$

If X is the observed number of successes in n independent Bernoulli trials in each of which the probability of success is p, and $Y = \frac{X}{n}$, then

$E(Y) = p$ and $\mathrm{Var}(Y) = \frac{p(1-p)}{n}$

For a random sample of n_x observations from $N(\mu_x, \sigma_x^2)$ and, independently, a random sample of n_y observations from $N(\mu_y, \sigma_y^2)$

$$\frac{(\bar{X} - \bar{Y}) - (\mu_x - \mu_y)}{\sqrt{\frac{\sigma_x^2}{n_x} + \frac{\sigma_y^2}{n_y}}} \sim N(0,1)$$

If $\sigma_x^2 = \sigma_y^2 = \sigma^2$ (unknown) then

$$\frac{(\bar{X} - \bar{Y}) - (\mu_x - \mu_y)}{\sqrt{S_p^2\left(\frac{1}{n_x} + \frac{1}{n_y}\right)}} \sim t_{n_x + n_y - 2} \quad \text{where } S_p^2 = \frac{(n_x - 1)S_x^2 + (n_y - 1)S_y^2}{n_x + n_y - 2}$$

Correlation and regression

For a set of n pairs of values (x_i, y_i)

$$S_{xx} = \Sigma(x_i - \overline{x})^2 = \Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}$$

$$S_{yy} = \Sigma(y_i - \overline{y})^2 = \Sigma y_i^2 - \frac{(\Sigma y_i)^2}{n}$$

$$S_{xy} = \Sigma(x_i - \overline{x})(y_i - \overline{y}) = \Sigma x_i y_i - \frac{(\Sigma x_i)(\Sigma y_i)}{n}$$

The product moment correlation coefficient is

$$r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = \frac{\Sigma(x_i - \overline{x})(y_i - \overline{y})}{\sqrt{\left\{\Sigma(x_i - \overline{x})^2\right\}\left\{\Sigma(y_i - \overline{y})^2\right\}}} = \frac{\Sigma x_i y_i - \frac{(\Sigma x_i)(\Sigma y_i)}{n}}{\sqrt{\left(\Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}\right)\left(\Sigma y_i^2 - \frac{(\Sigma y_i)^2}{n}\right)}}$$

The regression coefficient of y on x is $b = \frac{S_{xy}}{S_{xx}} = \frac{\Sigma(x_i - \overline{x})(y_i - \overline{y})}{\Sigma(x_i - \overline{x})^2}$

Least squares regression line of y on x is $y = a + bx$ where $a = \overline{y} - b\overline{x}$

Non-parametric tests

Goodness-of-fit test and contingency tables:

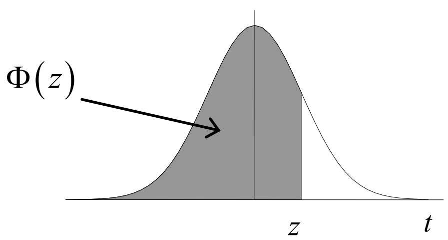
$$\sum \frac{(\mathrm{O}_i - \mathrm{E}_i)^2}{\mathrm{E}_i} \sim \chi^2_\nu$$

NORMAL PROBABILITY TABLE

Table of $\Phi(z)$

z													(ADD)					
	0.00	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359	4	8	12	16	20	24	28	32	36
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753	4	8	12	16	20	24	28	32	36
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141	4	8	12	15	19	23	27	31	35
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517	4	8	11	15	19	23	26	30	34
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879	4	7	11	14	18	22	25	29	32
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224	3	7	10	14	17	21	24	27	31
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549	3	6	10	13	16	19	23	26	29
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852	3	6	9	12	15	18	21	24	27
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133	3	6	8	11	14	17	19	22	25
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389	3	5	8	10	13	15	18	20	23
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621	2	5	7	9	12	14	16	18	21
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830	2	4	6	8	10	12	14	16	19
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015	2	4	6	7	9	11	13	15	16
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177	2	3	5	6	8	10	11	13	14
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319	1	3	4	6	7	8	10	11	13
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441	1	2	4	5	6	7	8	10	11
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545	1	2	3	4	5	6	7	8	9
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633	1	2	3	3	4	5	6	7	8
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706	1	1	2	3	4	4	5	6	6
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767	1	1	2	2	3	4	4	5	5
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817	0	1	1	2	2	3	3	4	4
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857	0	1	1	2	2	2	3	3	4
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890	0	1	1	1	2	2	2	3	3
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916	0	1	1	1	1	2	2	2	2
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936	0	0	1	1	1	1	1	2	2
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952	0	0	0	1	1	1	1	1	1
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964	0	0	0	0	1	1	1	1	1
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974	0	0	0	0	0	1	1	1	1
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981	0	0	0	0	0	0	0	1	1
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986	0	0	0	0	0	0	0	0	0
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990	0	0	0	0	0	0	0	0	0

The function tabulated is $\Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}t^2}\,\mathrm{d}t$. $\Phi(z)$ is the probability that a random variable having a Normal frequency density, with mean zero and variance unity, will be less than z.



BINOMIAL CUMULATIVE DISTRIBUTION FUNCTION

The tabulated value is $P(X \leq x)$, where X has a binomial distribution with index n and parameter p.

$p =$	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
$n = 5, x = 0$	0.7738	0.5905	0.4437	0.3277	0.2373	0.1681	0.1160	0.0778	0.0503	0.0313
1	0.9774	0.9185	0.8352	0.7373	0.6328	0.5282	0.4284	0.3370	0.2562	0.1875
2	0.9988	0.9914	0.9734	0.9421	0.8965	0.8369	0.7648	0.6826	0.5931	0.5000
3	1.0000	0.9995	0.9978	0.9933	0.9844	0.9692	0.9460	0.9130	0.8688	0.8125
4	1.0000	1.0000	0.9999	0.9997	0.9990	0.9976	0.9947	0.9898	0.9815	0.9688
$n = 6, x = 0$	0.7351	0.5314	0.3771	0.2621	0.1780	0.1176	0.0754	0.0467	0.0277	0.0156
1	0.9672	0.8857	0.7765	0.6554	0.5339	0.4202	0.3191	0.2333	0.1636	0.1094
2	0.9978	0.9842	0.9527	0.9011	0.8306	0.7443	0.6471	0.5443	0.4415	0.3438
3	0.9999	0.9987	0.9941	0.9830	0.9624	0.9295	0.8826	0.8208	0.7447	0.6563
4	1.0000	0.9999	0.9996	0.9984	0.9954	0.9891	0.9777	0.9590	0.9308	0.8906
5	1.0000	1.0000	1.0000	0.9999	0.9998	0.9993	0.9982	0.9959	0.9917	0.9844
$n = 7, x = 0$	0.6983	0.4783	0.3206	0.2097	0.1335	0.0824	0.0490	0.0280	0.0152	0.0078
1	0.9556	0.8503	0.7166	0.5767	0.4449	0.3294	0.2338	0.1586	0.1024	0.0625
2	0.9962	0.9743	0.9262	0.8520	0.7564	0.6471	0.5323	0.4199	0.3164	0.2266
3	0.9998	0.9973	0.9879	0.9667	0.9294	0.8740	0.8002	0.7102	0.6083	0.5000
4	1.0000	0.9998	0.9988	0.9953	0.9871	0.9712	0.9444	0.9037	0.8471	0.7734
5	1.0000	1.0000	0.9999	0.9996	0.9987	0.9962	0.9910	0.9812	0.9643	0.9375
6	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9994	0.9984	0.9963	0.9922
$n = 8, x = 0$	0.6634	0.4305	0.2725	0.1678	0.1001	0.0576	0.0319	0.0168	0.0084	0.0039
1	0.9428	0.8131	0.6572	0.5033	0.3671	0.2553	0.1691	0.1064	0.0632	0.0352
2	0.9942	0.9619	0.8948	0.7969	0.6785	0.5518	0.4278	0.3154	0.2201	0.1445
3	0.9996	0.9950	0.9786	0.9437	0.8862	0.8059	0.7064	0.5941	0.4770	0.3633
4	1.0000	0.9996	0.9971	0.9896	0.9727	0.9420	0.8939	0.8263	0.7396	0.6367
5	1.0000	1.0000	0.9998	0.9988	0.9958	0.9887	0.9747	0.9502	0.9115	0.8555
6	1.0000	1.0000	1.0000	0.9999	0.9996	0.9987	0.9964	0.9915	0.9819	0.9648
7	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9993	0.9983	0.9961
$n = 9, x = 0$	0.6302	0.3874	0.2316	0.1342	0.0751	0.0404	0.0207	0.0101	0.0046	0.0020
1	0.9288	0.7748	0.5995	0.4362	0.3003	0.1960	0.1211	0.0705	0.0385	0.0195
2	0.9916	0.9470	0.8591	0.7382	0.6007	0.4628	0.3373	0.2318	0.1495	0.0898
3	0.9994	0.9917	0.9661	0.9144	0.8343	0.7297	0.6089	0.4826	0.3614	0.2539
4	1.0000	0.9991	0.9944	0.9804	0.9511	0.9012	0.8283	0.7334	0.6214	0.5000
5	1.0000	0.9999	0.9994	0.9969	0.9900	0.9747	0.9464	0.9006	0.8342	0.7461
6	1.0000	1.0000	1.0000	0.9997	0.9987	0.9957	0.9888	0.9750	0.9502	0.9102
7	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9986	0.9962	0.9909	0.9805
8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9992	0.9980
$n = 10, x = 0$	0.5987	0.3487	0.1969	0.1074	0.0563	0.0282	0.0135	0.0060	0.0025	0.0010
1	0.9139	0.7361	0.5443	0.3758	0.2440	0.1493	0.0860	0.0464	0.0233	0.0107
2	0.9885	0.9298	0.8202	0.6778	0.5256	0.3828	0.2616	0.1673	0.0996	0.0547
3	0.9990	0.9872	0.9500	0.8791	0.7759	0.6496	0.5138	0.3823	0.2660	0.1719
4	0.9999	0.9984	0.9901	0.9672	0.9219	0.8497	0.7515	0.6331	0.5044	0.3770
5	1.0000	0.9999	0.9986	0.9936	0.9803	0.9527	0.9051	0.8338	0.7384	0.6230
6	1.0000	1.0000	0.9999	0.9991	0.9965	0.9894	0.9740	0.9452	0.8980	0.8281
7	1.0000	1.0000	1.0000	0.9999	0.9996	0.9984	0.9952	0.9877	0.9726	0.9453
8	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995	0.9983	0.9955	0.9893
9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9990

$p =$	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
$n = 12, x = 0$	0.5404	0.2824	0.1422	0.0687	0.0317	0.0138	0.0057	0.0022	0.0008	0.0002
1	0.8816	0.6590	0.4435	0.2749	0.1584	0.0850	0.0424	0.0196	0.0083	0.0032
2	0.9804	0.8891	0.7358	0.5583	0.3907	0.2528	0.1513	0.0834	0.0421	0.0193
3	0.9978	0.9744	0.9078	0.7946	0.6488	0.4925	0.3467	0.2253	0.1345	0.0730
4	0.9998	0.9957	0.9761	0.9274	0.8424	0.7237	0.5833	0.4382	0.3044	0.1938
5	1.0000	0.9995	0.9954	0.9806	0.9456	0.8822	0.7873	0.6652	0.5269	0.3872
6	1.0000	0.9999	0.9993	0.9961	0.9857	0.9614	0.9154	0.8418	0.7393	0.6128
7	1.0000	1.0000	0.9999	0.9994	0.9972	0.9905	0.9745	0.9427	0.8883	0.8062
8	1.0000	1.0000	1.0000	0.9999	0.9996	0.9983	0.9944	0.9847	0.9644	0.9270
9	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9992	0.9972	0.9921	0.9807
10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9989	0.9968
11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998
$n = 15, x = 0$	0.4633	0.2059	0.0874	0.0352	0.0134	0.0047	0.0016	0.0005	0.0001	0.0000
1	0.8290	0.5490	0.3186	0.1671	0.0802	0.0353	0.0142	0.0052	0.0017	0.0005
2	0.9638	0.8159	0.6042	0.3980	0.2361	0.1268	0.0617	0.0271	0.0107	0.0037
3	0.9945	0.9444	0.8227	0.6482	0.4613	0.2969	0.1727	0.0905	0.0424	0.0176
4	0.9994	0.9873	0.9383	0.8358	0.6865	0.5155	0.3519	0.2173	0.1204	0.0592
5	0.9999	0.9978	0.9832	0.9389	0.8516	0.7216	0.5643	0.4032	0.2608	0.1509
6	1.0000	0.9997	0.9964	0.9819	0.9434	0.8689	0.7548	0.6098	0.4522	0.3036
7	1.0000	1.0000	0.9994	0.9958	0.9827	0.9500	0.8868	0.7869	0.6535	0.5000
8	1.0000	1.0000	0.9999	0.9992	0.9958	0.9848	0.9578	0.9050	0.8182	0.6964
9	1.0000	1.0000	1.0000	0.9999	0.9992	0.9963	0.9876	0.9662	0.9231	0.8491
10	1.0000	1.0000	1.0000	1.0000	0.9999	0.9993	0.9972	0.9907	0.9745	0.9408
11	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995	0.9981	0.9937	0.9824
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9989	0.9963
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
$n = 20, x = 0$	0.3585	0.1216	0.0388	0.0115	0.0032	0.0008	0.0002	0.0000	0.0000	0.0000
1	0.7358	0.3917	0.1756	0.0692	0.0243	0.0076	0.0021	0.0005	0.0001	0.0000
2	0.9245	0.6769	0.4049	0.2061	0.0913	0.0355	0.0121	0.0036	0.0009	0.0002
3	0.9841	0.8670	0.6477	0.4114	0.2252	0.1071	0.0444	0.0160	0.0049	0.0013
4	0.9974	0.9568	0.8298	0.6296	0.4148	0.2375	0.1182	0.0510	0.0189	0.0059
5	0.9997	0.9887	0.9327	0.8042	0.6172	0.4164	0.2454	0.1256	0.0553	0.0207
6	1.0000	0.9976	0.9781	0.9133	0.7858	0.6080	0.4166	0.2500	0.1299	0.0577
7	1.0000	0.9996	0.9941	0.9679	0.8982	0.7723	0.6010	0.4159	0.2520	0.1316
8	1.0000	0.9999	0.9987	0.9900	0.9591	0.8867	0.7624	0.5956	0.4143	0.2517
9	1.0000	1.0000	0.9998	0.9974	0.9861	0.9520	0.8782	0.7553	0.5914	0.4119
10	1.0000	1.0000	1.0000	0.9994	0.9961	0.9829	0.9468	0.8725	0.7507	0.5881
11	1.0000	1.0000	1.0000	0.9999	0.9991	0.9949	0.9804	0.9435	0.8692	0.7483
12	1.0000	1.0000	1.0000	1.0000	0.9998	0.9987	0.9940	0.9790	0.9420	0.8684
13	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9985	0.9935	0.9786	0.9423
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9984	0.9936	0.9793
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9985	0.9941
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9987
17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$p =$	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
$n = 25, x = 0$	0.2774	0.0718	0.0172	0.0038	0.0008	0.0001	0.0000	0.0000	0.0000	0.0000
1	0.6424	0.2712	0.0931	0.0274	0.0070	0.0016	0.0003	0.0001	0.0000	0.0000
2	0.8729	0.5371	0.2537	0.0982	0.0321	0.0090	0.0021	0.0004	0.0001	0.0000
3	0.9659	0.7636	0.4711	0.2340	0.0962	0.0332	0.0097	0.0024	0.0005	0.0001
4	0.9928	0.9020	0.6821	0.4207	0.2137	0.0905	0.0320	0.0095	0.0023	0.0005
5	0.9988	0.9666	0.8385	0.6167	0.3783	0.1935	0.0826	0.0294	0.0086	0.0020
6	0.9998	0.9905	0.9305	0.7800	0.5611	0.3407	0.1734	0.0736	0.0258	0.0073
7	1.0000	0.9977	0.9745	0.8909	0.7265	0.5118	0.3061	0.1536	0.0639	0.0216
8	1.0000	0.9995	0.9920	0.9532	0.8506	0.6769	0.4668	0.2735	0.1340	0.0539
9	1.0000	0.9999	0.9979	0.9827	0.9287	0.8106	0.6303	0.4246	0.2424	0.1148
10	1.0000	1.0000	0.9995	0.9944	0.9703	0.9022	0.7712	0.5858	0.3843	0.2122
11	1.0000	1.0000	0.9999	0.9985	0.9893	0.9558	0.8746	0.7323	0.5426	0.3450
12	1.0000	1.0000	1.0000	0.9996	0.9966	0.9825	0.9396	0.8462	0.6937	0.5000
13	1.0000	1.0000	1.0000	0.9999	0.9991	0.9940	0.9745	0.9222	0.8173	0.6550
14	1.0000	1.0000	1.0000	1.0000	0.9998	0.9982	0.9907	0.9656	0.9040	0.7878
15	1.0000	1.0000	1.0000	1.0000	1.0000	0.9995	0.9971	0.9868	0.9560	0.8852
16	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9992	0.9957	0.9826	0.9461
17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9988	0.9942	0.9784
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9984	0.9927
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9980
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995
21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
$n = 30, x = 0$	0.2146	0.0424	0.0076	0.0012	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.5535	0.1837	0.0480	0.0105	0.0020	0.0003	0.0000	0.0000	0.0000	0.0000
2	0.8122	0.4114	0.1514	0.0442	0.0106	0.0021	0.0003	0.0000	0.0000	0.0000
3	0.9392	0.6474	0.3217	0.1227	0.0374	0.0093	0.0019	0.0003	0.0000	0.0000
4	0.9844	0.8245	0.5245	0.2552	0.0979	0.0302	0.0075	0.0015	0.0002	0.0000
5	0.9967	0.9268	0.7106	0.4275	0.2026	0.0766	0.0233	0.0057	0.0011	0.0002
6	0.9994	0.9742	0.8474	0.6070	0.3481	0.1595	0.0586	0.0172	0.0040	0.0007
7	0.9999	0.9922	0.9302	0.7608	0.5143	0.2814	0.1238	0.0435	0.0121	0.0026
8	1.0000	0.9980	0.9722	0.8713	0.6736	0.4315	0.2247	0.0940	0.0312	0.0081
9	1.0000	0.9995	0.9903	0.9389	0.8034	0.5888	0.3575	0.1763	0.0694	0.0214
10	1.0000	0.9999	0.9971	0.9744	0.8943	0.7304	0.5078	0.2915	0.1350	0.0494
11	1.0000	1.0000	0.9992	0.9905	0.9493	0.8407	0.6548	0.4311	0.2327	0.1002
12	1.0000	1.0000	0.9998	0.9969	0.9784	0.9155	0.7802	0.5785	0.3592	0.1808
13	1.0000	1.0000	1.0000	0.9991	0.9918	0.9599	0.8737	0.7145	0.5025	0.2923
14	1.0000	1.0000	1.0000	0.9998	0.9973	0.9831	0.9348	0.8246	0.6448	0.4278
15	1.0000	1.0000	1.0000	0.9999	0.9992	0.9936	0.9699	0.9029	0.7691	0.5722
16	1.0000	1.0000	1.0000	1.0000	0.9998	0.9979	0.9876	0.9519	0.8644	0.7077
17	1.0000	1.0000	1.0000	1.0000	0.9999	0.9994	0.9955	0.9788	0.9286	0.8192
18	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9986	0.9917	0.9666	0.8998
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9996	0.9971	0.9862	0.9506
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9991	0.9950	0.9786
21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9984	0.9919
22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9996	0.9974
23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9993
24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998
25	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$p =$	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
$n = 40, x = 0$	0.1285	0.0148	0.0015	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.3991	0.0805	0.0121	0.0015	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.6767	0.2228	0.0486	0.0079	0.0010	0.0001	0.0000	0.0000	0.0000	0.0000
3	0.8619	0.4231	0.1302	0.0285	0.0047	0.0006	0.0001	0.0000	0.0000	0.0000
4	0.9520	0.6290	0.2633	0.0759	0.0160	0.0026	0.0003	0.0000	0.0000	0.0000
5	0.9861	0.7937	0.4325	0.1613	0.0433	0.0086	0.0013	0.0001	0.0000	0.0000
6	0.9966	0.9005	0.6067	0.2859	0.0962	0.0238	0.0044	0.0006	0.0001	0.0000
7	0.9993	0.9581	0.7559	0.4371	0.1820	0.0553	0.0124	0.0021	0.0002	0.0000
8	0.9999	0.9845	0.8646	0.5931	0.2998	0.1110	0.0303	0.0061	0.0009	0.0001
9	1.0000	0.9949	0.9328	0.7318	0.4395	0.1959	0.0644	0.0156	0.0027	0.0003
10	1.0000	0.9985	0.9701	0.8392	0.5839	0.3087	0.1215	0.0352	0.0074	0.0011
11	1.0000	0.9996	0.9880	0.9125	0.7151	0.4406	0.2053	0.0709	0.0179	0.0032
12	1.0000	0.9999	0.9957	0.9568	0.8209	0.5772	0.3143	0.1285	0.0386	0.0083
13	1.0000	1.0000	0.9986	0.9806	0.8968	0.7032	0.4408	0.2112	0.0751	0.0192
14	1.0000	1.0000	0.9996	0.9921	0.9456	0.8074	0.5721	0.3174	0.1326	0.0403
15	1.0000	1.0000	0.9999	0.9971	0.9738	0.8849	0.6946	0.4402	0.2142	0.0769
16	1.0000	1.0000	1.0000	0.9990	0.9884	0.9367	0.7978	0.5681	0.3185	0.1341
17	1.0000	1.0000	1.0000	0.9997	0.9953	0.9680	0.8761	0.6885	0.4391	0.2148
18	1.0000	1.0000	1.0000	0.9999	0.9983	0.9852	0.9301	0.7911	0.5651	0.3179
19	1.0000	1.0000	1.0000	1.0000	0.9994	0.9937	0.9637	0.8702	0.6844	0.4373
20	1.0000	1.0000	1.0000	1.0000	0.9998	0.9976	0.9827	0.9256	0.7870	0.5627
21	1.0000	1.0000	1.0000	1.0000	1.0000	0.9991	0.9925	0.9608	0.8669	0.6821
22	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9970	0.9811	0.9233	0.7852
23	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9989	0.9917	0.9595	0.8659
24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9996	0.9966	0.9804	0.9231
25	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9988	0.9914	0.9597
26	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9996	0.9966	0.9808
27	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9988	0.9917
28	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9996	0.9968
29	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9989
30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997
31	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
32	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$p =$	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
$n = 50, x = 0$	0.0769	0.0052	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.2794	0.0338	0.0029	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.5405	0.1117	0.0142	0.0013	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.7604	0.2503	0.0460	0.0057	0.0005	0.0000	0.0000	0.0000	0.0000	0.0000
4	0.8964	0.4312	0.1121	0.0185	0.0021	0.0002	0.0000	0.0000	0.0000	0.0000
5	0.9622	0.6161	0.2194	0.0480	0.0070	0.0007	0.0001	0.0000	0.0000	0.0000
6	0.9882	0.7702	0.3613	0.1034	0.0194	0.0025	0.0002	0.0000	0.0000	0.0000
7	0.9968	0.8779	0.5188	0.1904	0.0453	0.0073	0.0008	0.0001	0.0000	0.0000
8	0.9992	0.9421	0.6681	0.3073	0.0916	0.0183	0.0025	0.0002	0.0000	0.0000
9	0.9998	0.9755	0.7911	0.4437	0.1637	0.0402	0.0067	0.0008	0.0001	0.0000
10	1.0000	0.9906	0.8801	0.5836	0.2622	0.0789	0.0160	0.0022	0.0002	0.0000
11	1.0000	0.9968	0.9372	0.7107	0.3816	0.1390	0.0342	0.0057	0.0006	0.0000
12	1.0000	0.9990	0.9699	0.8139	0.5110	0.2229	0.0661	0.0133	0.0018	0.0002
13	1.0000	0.9997	0.9868	0.8894	0.6370	0.3279	0.1163	0.0280	0.0045	0.0005
14	1.0000	0.9999	0.9947	0.9393	0.7481	0.4468	0.1878	0.0540	0.0104	0.0013
15	1.0000	1.0000	0.9981	0.9692	0.8369	0.5692	0.2801	0.0955	0.0220	0.0033
16	1.0000	1.0000	0.9993	0.9856	0.9017	0.6839	0.3889	0.1561	0.0427	0.0077
17	1.0000	1.0000	0.9998	0.9937	0.9449	0.7822	0.5060	0.2369	0.0765	0.0164
18	1.0000	1.0000	0.9999	0.9975	0.9713	0.8594	0.6216	0.3356	0.1273	0.0325
19	1.0000	1.0000	1.0000	0.9991	0.9861	0.9152	0.7264	0.4465	0.1974	0.0595
20	1.0000	1.0000	1.0000	0.9997	0.9937	0.9522	0.8139	0.5610	0.2862	0.1013
21	1.0000	1.0000	1.0000	0.9999	0.9974	0.9749	0.8813	0.6701	0.3900	0.1611
22	1.0000	1.0000	1.0000	1.0000	0.9990	0.9877	0.9290	0.7660	0.5019	0.2399
23	1.0000	1.0000	1.0000	1.0000	0.9996	0.9944	0.9604	0.8438	0.6134	0.3359
24	1.0000	1.0000	1.0000	1.0000	0.9999	0.9976	0.9793	0.9022	0.7160	0.4439
25	1.0000	1.0000	1.0000	1.0000	1.0000	0.9991	0.9900	0.9427	0.8034	0.5561
26	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9955	0.9686	0.8721	0.6641
27	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9981	0.9840	0.9220	0.7601
28	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9993	0.9924	0.9556	0.8389
29	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9966	0.9765	0.8987
30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9986	0.9884	0.9405
31	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9995	0.9947	0.9675
32	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9978	0.9836
33	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9991	0.9923
34	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9967
35	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9987
36	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9995
37	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998
38	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

POISSON CUMULATIVE DISTRIBUTION FUNCTION

The tabulated value is $P(X \leq x)$, where X has a Poisson distribution with parameter λ.

$\lambda =$	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
$x = 0$	0.6065	0.3679	0.2231	0.1353	0.0821	0.0498	0.0302	0.0183	0.0111	0.0067
1	0.9098	0.7358	0.5578	0.4060	0.2873	0.1991	0.1359	0.0916	0.0611	0.0404
2	0.9856	0.9197	0.8088	0.6767	0.5438	0.4232	0.3208	0.2381	0.1736	0.1247
3	0.9982	0.9810	0.9344	0.8571	0.7576	0.6472	0.5366	0.4335	0.3423	0.2650
4	0.9998	0.9963	0.9814	0.9473	0.8912	0.8153	0.7254	0.6288	0.5321	0.4405
5	1.0000	0.9994	0.9955	0.9834	0.9580	0.9161	0.8576	0.7851	0.7029	0.6160
6	1.0000	0.9999	0.9991	0.9955	0.9858	0.9665	0.9347	0.8893	0.8311	0.7622
7	1.0000	1.0000	0.9998	0.9989	0.9958	0.9881	0.9733	0.9489	0.9134	0.8666
8	1.0000	1.0000	1.0000	0.9998	0.9989	0.9962	0.9901	0.9786	0.9597	0.9319
9	1.0000	1.0000	1.0000	1.0000	0.9997	0.9989	0.9967	0.9919	0.9829	0.9682
10	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9990	0.9972	0.9933	0.9863
11	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9991	0.9976	0.9945
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9992	0.9980
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9993
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$\lambda =$	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
$x = 0$	0.0041	0.0025	0.0015	0.0009	0.0006	0.0003	0.0002	0.0001	0.0001	0.0000
1	0.0266	0.0174	0.0113	0.0073	0.0047	0.0030	0.0019	0.0012	0.0008	0.0005
2	0.0884	0.0620	0.0430	0.0296	0.0203	0.0138	0.0093	0.0062	0.0042	0.0028
3	0.2017	0.1512	0.1118	0.0818	0.0591	0.0424	0.0301	0.0212	0.0149	0.0103
4	0.3575	0.2851	0.2237	0.1730	0.1321	0.0996	0.0744	0.0550	0.0403	0.0293
5	0.5289	0.4457	0.3690	0.3007	0.2414	0.1912	0.1496	0.1157	0.0885	0.0671
6	0.6860	0.6063	0.5265	0.4497	0.3782	0.3134	0.2562	0.2068	0.1649	0.1301
7	0.8095	0.7440	0.6728	0.5987	0.5246	0.4530	0.3856	0.3239	0.2687	0.2202
8	0.8944	0.8472	0.7916	0.7291	0.6620	0.5925	0.5231	0.4557	0.3918	0.3328
9	0.9462	0.9161	0.8774	0.8305	0.7764	0.7166	0.6530	0.5874	0.5218	0.4579
10	0.9747	0.9574	0.9332	0.9015	0.8622	0.8159	0.7634	0.7060	0.6453	0.5830
11	0.9890	0.9799	0.9661	0.9467	0.9208	0.8881	0.8487	0.8030	0.7520	0.6968
12	0.9955	0.9912	0.9840	0.9730	0.9573	0.9362	0.9091	0.8758	0.8364	0.7916
13	0.9983	0.9964	0.9929	0.9872	0.9784	0.9658	0.9486	0.9261	0.8981	0.8645
14	0.9994	0.9986	0.9970	0.9943	0.9897	0.9827	0.9726	0.9585	0.9400	0.9165
15	0.9998	0.9995	0.9988	0.9976	0.9954	0.9918	0.9862	0.9780	0.9665	0.9513
16	0.9999	0.9998	0.9996	0.9990	0.9980	0.9963	0.9934	0.9889	0.9823	0.9730
17	1.0000	0.9999	0.9998	0.9996	0.9992	0.9984	0.9970	0.9947	0.9911	0.9857
18	1.0000	1.0000	0.9999	0.9999	0.9997	0.9993	0.9987	0.9976	0.9957	0.9928
19	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9995	0.9989	0.9980	0.9965
20	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9991	0.9984
21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9993
22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9997

PERCENTAGE POINTS OF THE χ^2 DISTRIBUTION

The values in the table are those which a random variable with the χ^2 distribution on ν degrees of freedom exceeds with the probability shown.

ν	0.995	0.990	0.975	0.950	0.900	0.100	0.050	0.025	0.010	0.005
1	0.000	0.000	0.001	0.004	0.016	2.705	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.070	12.832	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.580	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.300
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	14.042	30.813	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.558
25	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	18.114	36.741	40.113	43.194	46.963	49.645
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993
29	13.121	14.256	16.047	17.708	19.768	39.088	42.557	45.722	49.588	52.336
30	13.787	14.953	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672

PERCENTAGE POINTS OF STUDENT'S *t* DISTRIBUTION

The values in the table are those which a random variable with student's t distribution on v degrees of freedom exceeds with the probability shown.

v	0.10	0.05	0.025	0.01	0.005
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
32	1.309	1.694	2.037	2.449	2.738
34	1.307	1.691	2.032	2.441	2.728
36	1.306	1.688	2.028	2.435	2.719
38	1.304	1.686	2.024	2.429	2.712
40	1.303	1.684	2.021	2.423	2.704
45	1.301	1.679	2.014	2.412	2.690
50	1.299	1.676	2.009	2.403	2.678
55	1.297	1.673	2.004	2.396	2.668
60	1.296	1.671	2.000	2.390	2.660
70	1.294	1.667	1.994	2.381	2.648
80	1.292	1.664	1.990	2.374	2.639
90	1.291	1.662	1.987	2.369	2.632
100	1.290	1.660	1.984	2.364	2.626
110	1.289	1.659	1.982	2.361	2.621
120	1.289	1.658	1.980	2.358	2.617

CRITICAL VALUES FOR CORRELATION COEFFICIENTS

These tables concern tests of the hypothesis that a population correlation coefficient ρ is 0. The values in the tables are the minimum values which need to be reached by a sample correlation coefficient in order to be significant at the level shown, on a one-tailed test.

Sample	Product Moment Coefficient				
			Level		
Level	0.10	0.05	0.025	0.01	0.005
4	0.8000	0.9000	0.9500	0.9800	0.9900
5	0.6870	0.8054	0.8783	0.9343	0.9587
6	0.6084	0.7293	0.8114	0.8822	0.9172
7	0.5509	0.6694	0.7545	0.8329	0.8745
8	0.5067	0.6215	0.7067	0.7887	0.8343
9	0.4716	0.5822	0.6664	0.7498	0.7977
10	0.4428	0.5494	0.6319	0.7155	0.7646
11	0.4187	0.5214	0.6021	0.6851	0.7348
12	0.3981	0.4973	0.5760	0.6581	0.7079
13	0.3802	0.4762	0.5529	0.6339	0.6835
14	0.3646	0.4575	0.5324	0.6120	0.6614
15	0.3507	0.4409	0.5140	0.5923	0.6411
16	0.3383	0.4259	0.4973	0.5742	0.6226
17	0.3271	0.4124	0.4821	0.5577	0.6055
18	0.3170	0.4000	0.4683	0.5425	0.5897
19	0.3077	0.3887	0.4555	0.5285	0.5751
20	0.2992	0.3783	0.4438	0.5155	0.5614
21	0.2914	0.3687	0.4329	0.5034	0.5487
22	0.2841	0.3598	0.4227	0.4921	0.5368
23	0.2774	0.3515	0.4133	0.4815	0.5256
24	0.2711	0.3438	0.4044	0.4716	0.5151
25	0.2653	0.3365	0.3961	0.4622	0.5052
26	0.2598	0.3297	0.3882	0.4534	0.4958
27	0.2546	0.3233	0.3809	0.4451	0.4869
28	0.2497	0.3172	0.3739	0.4372	0.4785
29	0.2451	0.3115	0.3673	0.4297	0.4705
30	0.2407	0.3061	0.3610	0.4226	0.4629
40	0.2070	0.2638	0.3120	0.3665	0.4026
50	0.1843	0.2353	0.2787	0.3281	0.3610
60	0.1678	0.2144	0.2542	0.2997	0.3301
70	0.1550	0.1982	0.2352	0.2776	0.3060
80	0.1448	0.1852	0.2199	0.2597	0.2864
90	0.1364	0.1745	0.2072	0.2449	0.2702
100	0.1292	0.1654	0.1966	0.2324	0.2565

Cycle indices for 3D rotational symmetry groups acting on:

Vertices of a Tetrahedron	$\frac{1}{12}(x_1^4 + 8x_1^1x_3^1 + 3x_2^2)$
Faces of a Tetrahedron	$\frac{1}{12}(x_1^4 + 8x_1^1x_3^1 + 3x_2^2)$
Edges of a Tetrahedron	$\frac{1}{12}(x_1^6 + 8x_3^2 + 3x_1^2x_2^2)$
Vertices of a Cube	$\frac{1}{24}(x_1^8 + 8x_1^2x_3^2 + 9x_2^4 + 6x_4^2)$
Faces of a Cube	$\frac{1}{24}(x_1^6 + 6x_1^2x_4^1 + 3x_1^2x_2^2 + 6x_2^3 + 8x_3^2)$
Edges of a Cube	$\frac{1}{24}\left(x_1^{12} + 3x_2^6 + 6x_1^2x_2^5 + 6x_4^3 + 8x_3^4\right)$
Vertices of an Octahedron	$\frac{1}{24}(x_1^6 + 6x_1^2x_4^1 + 3x_1^2x_2^2 + 6x_2^3 + 8x_3^2)$
Faces of an Octahedron	$\frac{1}{24}(x_1^8 + 8x_1^2x_3^2 + 9x_2^4 + 6x_4^2)$
Edges of an Octahedron	$\frac{1}{24}\left(x_1^{12} + 3x_2^6 + 6x_1^2x_2^5 + 6x_4^3 + 8x_3^4\right)$

Cycle Indices for 3D rotational symmetry groups (rotation plus flip) acting on polygons.

Polygon with p (prime) vertices	$\frac{1}{2p}\left(x_1^p + (p-1)x_p^1 + px_1^1x_2^{\frac{p-1}{2}}\right)$
Square	$\frac{1}{8}(x_1^4 + 3x_2^2 + 2x_4^1 + 2x_1^2x_2^1)$
Hexagon	$\frac{1}{12}(x_1^6 + 4x_2^3 + 2x_3^2 + 2x_6^1 + 3x_1^2x_2^2)$
Octagon	$\frac{1}{16}(x_1^8 + 4x_1^2x_2^3 + 5x_2^4 + 2x_4^2 + 4x_8^1)$
Nonagon	$\frac{1}{18}(x_1^9 + 9x_1^1x_2^4 + 6x_9^1 + 2x_3^3)$
Decagon	$\frac{1}{20}\left(x_1^{10} + 6x_2^5 + 4x_5^2 + 4x_{10}^1 + 5x_1^2x_2^4\right)$
Dodecagon	$\frac{1}{24}\left(x_1^{12} + 7x_2^6 + 2x_3^4 + 2x_4^3 + 2x_6^2 + 4x_{12}^1 + 6x_1^2x_2^5\right)$